2. A second, more precise, visual check begins with graphing $y_1 = x^3 + 2$ and $y_2 = \sqrt[3]{x - 2}$ using a squared viewing window. Then press DRAW 8 VARS ▷ 1 1 ENTER to select the DrawInv option of the DRAW menu and graph the inverse. The resulting graph should coincide with the graph of y_2.
3. For a third check, note that if y_2 is the inverse of y_1, then $(y_2 \circ y_1)(x) = x$ and $(y_1 \circ y_2)(x) = x$. Enter $y_3 = y_2(y_1(x))$ and $y_4 = y_1(y_2(x))$, and form a table to compare x, y_3, and y_4. Note that for the values shown, $y_3 = y_4 = x$.

TblStart = −3, ΔTbl = 1

X	Y3	Y4
−3	−3	−3
−2	−2	−2
−1	−1	−1
0	0	0
1	1	1
2	2	2
3	3	3

X = −3

Your Turn

1. Enter $y_1 = 2x + 1$ and $y_2 = x/2 - 1$. These are *not* inverse functions, as the following three activities will demonstrate.
2. Graph y_1 and y_2 in a squared viewing window. These should not appear to be reflections across the line $y = x$.
3. From the home screen, press DRAW 8 VARS ▷ 1 1 ENTER. The graph drawn should be different from the graph of y_2.
4. Enter $y_3 = y_2(y_1(x))$ and $y_4 = y_1(y_2(x))$ and form a table. Compare x, y_3, and y_4. The columns should *not* be the same.

9.1 Exercise Set

FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *Classify each of the following statements as either true or false.*

1. The composition of two functions f and g is written $f \circ g$. True
2. The notation $(f \circ g)(x)$ means $f(g(x))$. True
3. If $f(x) = x^2$ and $g(x) = x + 3$, then $(g \circ f)(x) = (x + 3)^2$. False
4. For any function h, there is only one way to decompose the function as $h = f \circ g$. False
5. The function f is one-to-one if $f(1) = 1$. False
6. The -1 in f^{-1} is an exponent. False
7. The function f is the inverse of f^{-1}. True
8. If g and h are inverses of each other, then $(g \circ h)(x) = x$. True

For each pair of functions, find **(a)** $(f \circ g)(1)$; **(b)** $(g \circ f)(1)$; **(c)** $(f \circ g)(x)$; *and* **(d)** $(g \circ f)(x)$.

9. $f(x) = x^2 + 1;\ g(x) = x - 3$ ⊡
10. $f(x) = x + 4;\ g(x) = x^2 - 5$ ⊡
11. $f(x) = 5x + 1;\ g(x) = 2x^2 - 7$ ⊡
12. $f(x) = 3x^2 + 4;\ g(x) = 4x - 1$ ⊡

⊡ Answers to Exercises 9–12 are on p. IA-21.

13. $f(x) = x + 7;\ g(x) = 1/x^2$ ⊡

14. $f(x) = 1/x^2;\ g(x) = x + 2$ ⊡

15. $f(x) = \sqrt{x};\ g(x) = x + 3$ ⊡

16. $f(x) = 10 - x;\ g(x) = \sqrt{x}$ ⊡

17. $f(x) = \sqrt{4x};\ g(x) = 1/x$ ⊡

18. $f(x) = \sqrt{x + 3};\ g(x) = 13/x$ ⊡

19. $f(x) = x^2 + 4;\ g(x) = \sqrt{x - 1}$ ⊡

20. $f(x) = x^2 + 8;\ g(x) = \sqrt{x + 17}$ ⊡

Use the following table to find each value, if possible.

X	Y1	Y2
−3	−4	1
−2	−1	−2
−1	2	−3
0	5	−2
1	8	1
2	11	6
3	14	11
X =		

21. $(y_1 \circ y_2)(-3)$ 8

22. $(y_2 \circ y_1)(-3)$ Not defined

23. $(y_1 \circ y_2)(-1)$ −4

24. $(y_2 \circ y_1)(-1)$ 6

25. $(y_2 \circ y_1)(1)$ Not defined

26. $(y_1 \circ y_2)(1)$ 8

Use the table below to find each value, if possible.

x	$f(x)$	$g(x)$
1	0	1
2	3	5
3	2	8
4	6	5
5	4	1

27. $(f \circ g)(2)$ 4

28. $(g \circ f)(4)$ Not defined

29. $f(g(3))$ Not defined

30. $g(f(5))$ 5

Find $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$. Answers may vary.

31. $h(x) = (3x - 5)^4$ ⊡

32. $h(x) = (2x + 7)^3$ ⊡

33. $h(x) = \sqrt{2x + 7}$ ⊡

34. $h(x) = \sqrt[3]{4x - 5}$ ⊡

35. $h(x) = \dfrac{2}{x - 3}$ ⊡

36. $h(x) = \dfrac{3}{x} + 4$ ⊡

37. $h(x) = \dfrac{1}{\sqrt{7x + 2}}$ ⊡

38. $h(x) = \sqrt{x - 7} - 3$ ⊡

39. $h(x) = \dfrac{1}{\sqrt{3x}} + \sqrt{3x}$ ⊡

40. $h(x) = \dfrac{1}{\sqrt{2x}} - \sqrt{2x}$ ⊡

Determine whether each function is one-to-one.

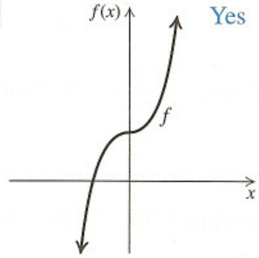
41. $f(x) = x - 5$ Yes

42. $f(x) = 5 - 2x$ Yes

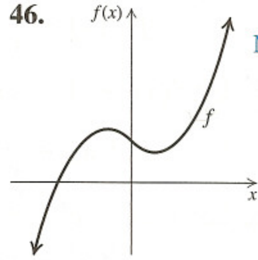
43. $f(x) = x^2 + 1$ No

44. $f(x) = 1 - x^2$ No

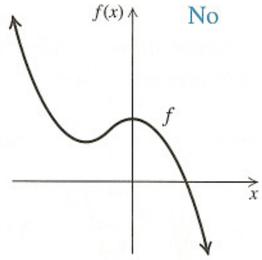
45. Yes



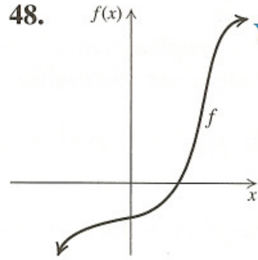
46. No



47. No



48. Yes



*For each function, **(a)** determine whether it is one-to-one and **(b)** if it is one-to-one, find a formula for the inverse.*

49. $f(x) = x + 4$ **(a)** Yes; **(b)** $f^{-1}(x) = x - 4$

50. $f(x) = x + 2$ **(a)** Yes; **(b)** $f^{-1}(x) = x - 2$

51. $f(x) = 2x$ **(a)** Yes; **(b)** $f^{-1}(x) = x/2$

52. $f(x) = 3x$ **(a)** Yes; **(b)** $f^{-1}(x) = x/3$

53. $g(x) = 3x - 1$ ⊡

54. $g(x) = 2x - 5$ ⊡

55. $f(x) = \frac{1}{2}x + 1$ **(a)** Yes; **(b)** $f^{-1}(x) = 2x - 2$

56. $f(x) = \frac{1}{3}x + 2$ **(a)** Yes; **(b)** $f^{-1}(x) = 3x - 6$

57. $g(x) = x^2 + 5$ **(a)** No

58. $g(x) = x^2 - 4$ **(a)** No

59. $h(x) = -10 - x$ ⊡

60. $h(x) = 7 - x$ ⊡

Aha! **61.** $f(x) = \dfrac{1}{x}$ ⊡

62. $f(x) = \dfrac{3}{x}$ ⊡

63. $G(x) = 4$ **(a)** No

64. $H(x) = 2$ **(a)** No

65. $f(x) = \dfrac{2x + 1}{3}$ ⊡

66. $f(x) = \dfrac{3x + 2}{5}$ ⊡

67. $f(x) = x^3 - 5$ ⊡

68. $f(x) = x^3 + 7$ ⊡

69. $g(x) = (x - 2)^3$ ⊡

70. $g(x) = (x + 7)^3$ ⊡

71. $f(x) = \sqrt{x}$ ⊡

72. $f(x) = \sqrt{x - 1}$ ⊡

Graph each function and its inverse using the same set of axes.

73. $f(x) = \frac{2}{3}x + 4$ ⊡

74. $g(x) = \frac{1}{4}x + 2$ ⊡

75. $f(x) = x^3 + 1$ ⊡

76. $f(x) = x^3 - 1$ ⊡

77. $g(x) = \frac{1}{2}x^3$ ⊡

78. $g(x) = \frac{1}{3}x^3$ ⊡

79. $F(x) = -\sqrt{x}$ ⊡

80. $f(x) = \sqrt{x}$ ⊡

81. $f(x) = -x^2, x \geq 0$ ⊡

82. $f(x) = x^2 - 1, x \leq 0$ ⊡

⊡ Answers to Exercises 13–20, 31–40, 53, 54, 59–62, and 65–82 are on pp. IA-21 and IA-22.

83. Let $f(x) = \sqrt[3]{x - 4}$. Use composition to show that $f^{-1}(x) = x^3 + 4$. ⊡

84. Let $f(x) = 3/(x + 2)$. Use composition to show that
$$f^{-1}(x) = \frac{3}{x} - 2.$$ ⊡

85. Let $f(x) = (1 - x)/x$. Use composition to show that
$$f^{-1}(x) = \frac{1}{x + 1}.$$ ⊡

86. Let $f(x) = x^3 - 5$. Use composition to show that $f^{-1}(x) = \sqrt[3]{x + 5}$. ⊡

Use a graphing calculator to help determine whether or not the given pairs of functions are inverses of each other.

87. $f(x) = 0.75x^2 + 2$; $g(x) = \sqrt{\dfrac{4(x - 2)}{3}}$ No

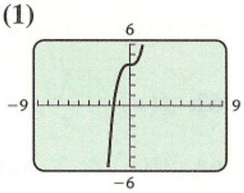
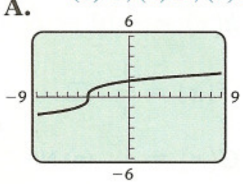
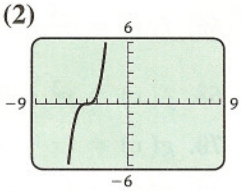
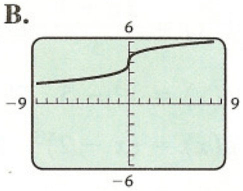
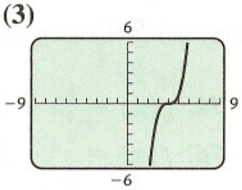
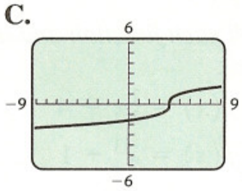
88. $f(x) = 1.4x^3 + 3.2$; $g(x) = \sqrt[3]{\dfrac{x - 3.2}{1.4}}$ Yes

89. $f(x) = \sqrt{2.5x + 9.25}$;
$g(x) = 0.4x^2 - 3.7, x \geq 0$ Yes

90. $f(x) = 0.8x^{1/2} + 5.23$;
$g(x) = 1.25(x^2 - 5.23), x \geq 0$ No

In Exercises 91 and 92, match the graph of each function in Column A with the graph of its inverse in Column B.

91. Column A | Column B
(1) C; (2) D; (3) B; (4) A

(1)

A.

(2)

B.

(3)

C.

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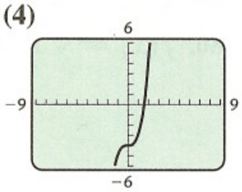
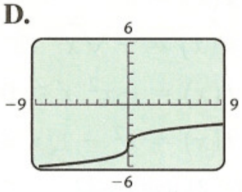
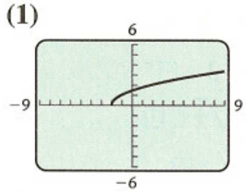
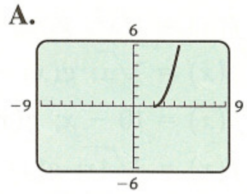
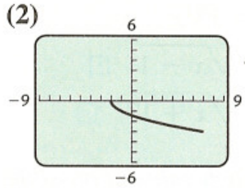
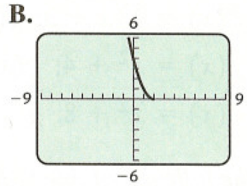

D.


⊡ Answers to Exercises 83–86 are on p. IA-22.

92. Column A | Column B

(1)

A.

(2)

B.

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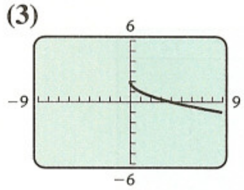
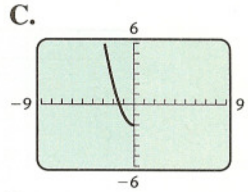
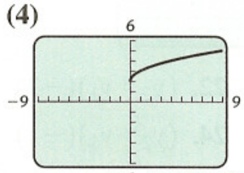
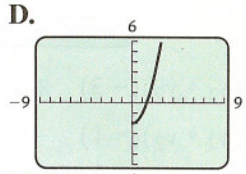

C.

(4)

D.


(1) D; (2) C; (3) B; (4) A

93. *Dress Sizes in the United States and France.* A size-6 dress in the United States is size 38 in France. A function that converts dress sizes in the United States to those in France is
$$f(x) = x + 32.$$

a) Find the dress sizes in France that correspond to sizes 8, 10, 14, and 18 in the United States. 40, 42, 46, 50

b) Determine whether this function has an inverse that is a function. If so, find a formula for the inverse. Yes; $f^{-1}(x) = x - 32$

c) Use the inverse function to find dress sizes in the United States that correspond to sizes 40, 42, 46, and 50 in France. 8, 10, 14, 18

94. *Dress Sizes in the United States and Italy.* A size-6 dress in the United States is size 36 in Italy. A function that converts dress sizes in the United States to those in Italy is
$$f(x) = 2(x + 12).$$

a) Find the dress sizes in Italy that correspond to sizes 8, 10, 14, and 18 in the United States. 40, 44, 52, 60

b) Determine whether this function has an inverse that is a function. If so, find a formula for the inverse. Yes; $f^{-1}(x) = (x/2) - 12$

c) Use the inverse function to find dress sizes in the United States that correspond to sizes 40, 44, 52, and 60 in Italy. 8, 10, 14, 18

TW **95.** Is there a one-to-one relationship between items in a store and the price of each of those items? Why or why not?

TW **96.** Mathematicians usually try to select "logical" words when forming definitions. Does the term "one-to-one" seem logical? Why or why not?

SKILL REVIEW

To prepare for Section 9.2, review simplifying exponential expressions and graphing equations (Sections 1.4, 1.5, and 7.2).

Simplify.

97. 2^{-3} [1.4] $\frac{1}{8}$

98. $5^{(1-3)}$ [1.4] $\frac{1}{25}$

99. $4^{5/2}$ [7.2] 32

100. $3^{7/10}$ [7.2] Approximately 2.1577

Graph. [1.5]

101. $y = x^3$ ⊡

102. $x = y^3$ ⊡

SYNTHESIS

TW **103.** The function $V(t) = 750(1.2)^t$ is used to predict the value, $V(t)$, of a certain rare stamp t years from 2005. Do not calculate $V^{-1}(t)$, but explain how V^{-1} could be used.

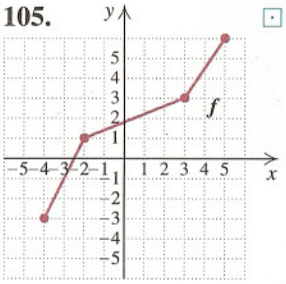
TW **104.** An organization determines that the cost per person of chartering a bus is given by the function

$$C(x) = \frac{100 + 5x}{x},$$

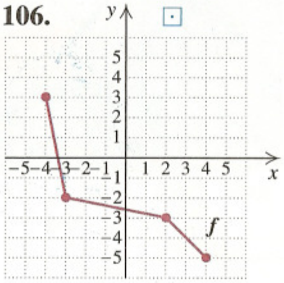
where x is the number of people in the group and $C(x)$ is in dollars. Determine $C^{-1}(x)$ and explain how this inverse function could be used.

For Exercises 105 and 106, graph the inverse of f.

105. ⊡



106. ⊡



107. *Dress Sizes in France and Italy.* Use the information in Exercises 93 and 94 to find a function for the French dress size that corresponds to a size x dress in Italy. $g(x) = \frac{x}{2} + 20$

108. *Dress Sizes in Italy and France.* Use the information in Exercises 93 and 94 to find a function for the Italian dress size that corresponds to a size x dress in France. $h(x) = 2(x - 20)$

TW **109.** What relationship exists between the answers to Exercises 107 and 108? Explain how you determined this.

110. Show that function composition is associative by showing that $((f \circ g) \circ h)(x) = (f \circ (g \circ h))(x)$. ⊡

111. Show that if $h(x) = (f \circ g)(x)$, then $h^{-1}(x) = (g^{-1} \circ f^{-1})(x)$. (*Hint*: Use Exercise 110.) ⊡

112. Match each function in Column A with its inverse from Column B.

Column A		Column B
(1) $y = 5x^3 + 10$	C	A. $y = \frac{\sqrt[3]{x} - 10}{5}$
(2) $y = (5x + 10)^3$	A	B. $y = \sqrt[3]{\frac{x}{5}} - 10$
(3) $y = 5(x + 10)^3$	B	C. $y = \sqrt[3]{\frac{x - 10}{5}}$
(4) $y = (5x)^3 + 10$	D	D. $y = \frac{\sqrt[3]{x - 10}}{5}$

TW **113.** Examine the following table. Is it possible that f and g could be inverses of each other? Why or why not?

x	$f(x)$	$g(x)$
6	6	6
7	6.5	8
8	7	10
9	7.5	12
10	8	14
11	8.5	16
12	9	18

114. Assume in Exercise 113 that f and g are both linear functions. Find equations for $f(x)$ and $g(x)$ Are f and g inverses of each other? $f(x) = \frac{1}{2}x + 3$; $g(x) = 2x - 6$; yes

115. Let $c(w)$ represent the cost of mailing a package that weighs w pounds. Let $f(n)$ represent the weight, in pounds, of n copies of a certain book. Explain what $(c \circ f)(n)$ represents. The cost of mailing n copies of the book

116. Let $g(a)$ represent the number of gallons of sealant needed to seal a bamboo floor with area a. Let $c(s)$ represent the cost of s gallons of sealant. Which composition makes sense: $(c \circ g)(a)$ or $(g \circ c)(s)$? What does it represent? ⊡

⊡ Answers to Exercises 101, 102, 105, 106, 110, 111, and 116 are on pp. IA-22 and IA-23.